

POTENTIAL EVAPOTRANSPIRATION: A SIMPLIFICATION OF THORNTHWAITE'S METHOD

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Abstract

It is shown that, where mean monthly temperatures do not fall below freezing point, the results which are obtained by Thornthwaite's method for calculating Potential Evapotranspiration may be exactly reproduced by a simple graphical method.

Introduction

The data available on the various climatic elements show very great differences in the density of the observing net-work and in the length of record. For only a few elements can the observations be regarded as reasonably complete. If the results of detailed special studies are to be extended beyond the area originally examined this can usually be done only if empirical formulae can be established which relate them to elements for which wide-spread observations are available. It is evident that the validity of such formulae outside the area for which they were devised can never be assumed.

Thornthwaite (1944, 1948) has proposed an empirical method for computing from temperature records the potential evapotranspiration, defined as 'the water-loss that would occur if soil moisture were constantly at the optimum level for plant growth.' As originally published the method offered to water-supply engineers and agriculturists empirical equations which fitted the observed water-loss in various American localities where water-need was fully met. In 1948 Thornthwaite gave his method a more general significance by proposing that potential evapotranspiration should be used as an essential element in a world classification of climate promising escape from many of the logical difficulties of earlier classifications.

It is therefore desirable that the method should be fully tested outside the area for which it was devised. One approach is for ecologists, engineers and others concerned with water-need to examine the extent to which the results obtained by Thornthwaite's method are consistent with their own observations. Unfortunately, as Thornthwaite is the first to admit, the method proposed for computing potential evapotranspiration is clumsy. The awkwardness of the method impedes theoretical discussion of its implications and involves the danger that workers who might contribute to its practical testing may be discouraged from doing so.

The aim of the present paper is to show that, under Australian conditions, the results which Thornthwaite's method give may be obtained far more simply. Throughout the discussion the aim is to reproduce the results obtained by the application of the original formulae. Since Thornthwaite claims only that his method gives values which are 'approximately correct' he presumably regards it as a matter of arithmetical convenience that he quotes results to hundredths of an inch or, in one table (1948, fig. 13) to hundredths of a centimeter. For the same reason figures are given in the present paper to an 'accuracy' which can have no physical significance.

Thornthwaite's Method

(The following discussion follows Thornthwaite in writing PE for potential evapotranspiration.)

Three steps are required:

(1) Determine for the station the annual heat index I , by summing 12 monthly heat indexes $i = (t/5)^{1.514}$, where $t^\circ\text{C}$ is the mean monthly temperature.

(2) Use I to determine the slope of a straight line on a plot of $\log PE$ against $\log t$ (Thornthwaite 1948, fig. 13). This is drawn to pass through the appropriate point on an I scale on the graph, actually the point at which $t = (I/10)^\circ\text{C}$ and $PE = 1.6\text{ cm.}$, and the 'point of convergence' at which $t = 26.5^\circ\text{C}$ and $PE = 13.5\text{ cm.}$ Read from the graph the value of PE corresponding to the monthly values of t . (The same result may be obtained from the equations given (Thornthwaite 1946, 1948), but these equations are obviously derived from straight lines fitted to plotted values of $\log PE$ against $\log t$. The graphical form of the calculation is thus the original form though it was published later.)

(3) Multiply the unadjusted values of PE obtained in Step 2 by a factor (tabulated in Thornthwaite 1948, Table IV) to adjust for differences in the length of months and for seasonal changes in duration of daylight.

In the present paper the PE - t relationship has been replotted for a range of values of I . The use of linear scales on the axes loses the advantages of the straight lines of the log-log graph but probably gives a clearer view of the relationship between PE and t . The curves, drawn for values of $I = 30, 40, \dots, 120$, are closely enough spaced to permit accurate interpolation of those corresponding to intermediate values of I .

Simplification

It is evident that the calculations can be simplified only if a way can be found to avoid the computation of I . If it can be shown that I may be determined with sufficient accuracy from the annual mean temperature and range it becomes possible to add to the plotted curves (either the logarithmic or the linear version) a set of subsidiary curves which will permit the selection of the appropriate PE - t curve without a preliminary calculation of I .

It is clear that Thornthwaite's method, designed for climates where some or many months have temperatures below 0°C , must obtain I by summation, but under Australian conditions this may be avoided. Since any method of expressing I as a function of mean and range must involve assumptions about the form of annual temperature curves the ultimate justification of any such method must be empirical. The method used in arriving at the rule used here is thus an indication of line of approach rather than a proof and is presented in outline as an Appendix.

The formula proposed is:

$$I = i_m (12 + .6 r^2/m^2)$$

where $m^\circ\text{C}$ = annual mean temperature,
 $r^\circ\text{C}$ = difference between mean temperature of hottest and
 coldest months,
 and $i_m = (m/5)^{1.514}$ = i value for a temperature equal to annual
 mean.

The results of applying this formula have been compared with those obtained by summing 12 monthly values of i for many stations. Examples are given in Table 1. It will be seen that the differences are quite negligible even in the case of stations having up to three months below 0°C .

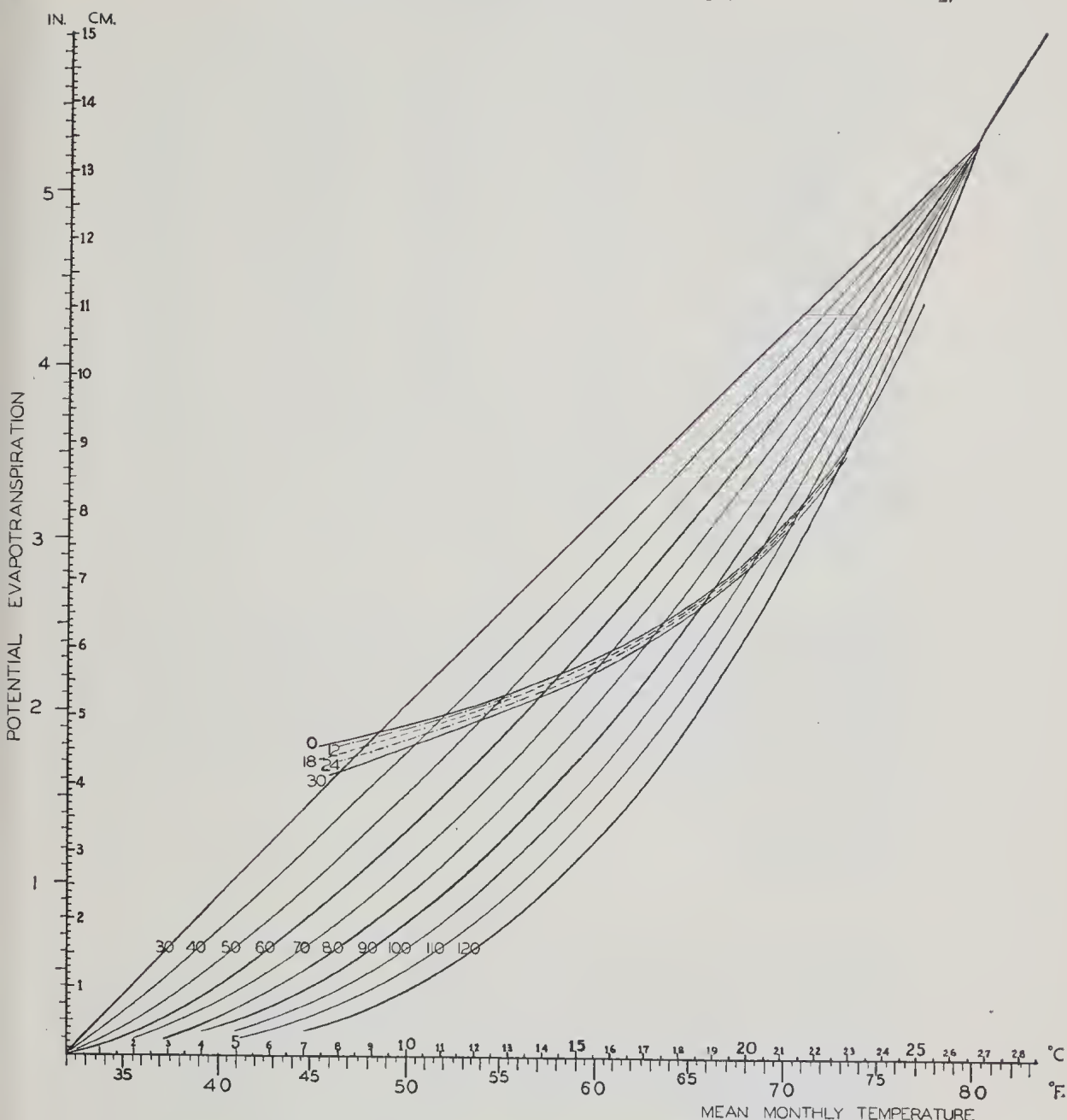


FIG. 1.—Graphs for obtaining Thornthwaite's values for unadjusted potential evapotranspiration. The curves numbered 30 . . . 120 give relationship of monthly temperature and monthly PE for selected values of annual heat index. The curves crossing them, numbered 0-30, give annual range of temp. in degrees F.

Determine annual mean temp. and range. Draw line perpendicular to temp. axis through annual mean temperature. The point at which this crosses the appropriate range curves lies on the *PE-t* graph for the station which may then be drawn with sufficient accuracy by interpolation between the *PE-t* curves provided. Read the unadjusted values of PE corresponding to the monthly mean temperatures.

TABLE 1

Station	$m(^{\circ}\text{C})$	$r(^{\circ}\text{C})$	I from rule	$I = \Sigma i$
Alice Springs	20.9	16.8	108.0	108.07
Brisbane	20.5	10.4	103.0	102.92
Perth	18.0	10.4	84.8	84.77
Sydney	17.3	10.4	103.0	102.92
Adelaide	17.2	12.2	79.8	79.96
Merbein	16.7	14.5	77.3	77.01
Melbourne	14.7	10.5	63.0	62.91
Hobart	12.4	9.0	48.8	48.66
Berlin	8.5	18.5	33.1	34.0
Pittsburgh	11.5	24.4	51.9	52.96
Omaha	10.1	31.1	50.5	52.8
Chicago	9.2	27.0	43.3	44.6

Using this formula it is possible to compute I and hence PE for a wide range of values of m and r . For example, when $m = 10^{\circ}\text{C}$ the unadjusted values of PE corresponding to various ranges are:

Range (degrees C)	0	3.3	6.7	10.0	13.3	16.7
(degrees F)	0	6	12	18	24	30
PE (cm.)	4.88	4.87	4.83	4.76	4.67	4.56

Using sufficient calculated values of this sort curves have been drawn crossing the $PE-t$ curves. These are marked with the range (in degrees F) to which each corresponds and permit the selection or interpolation of the appropriate $PE-t$ curve without the calculation of I . Thus if a station has an annual mean of 60°F and a range of 18°F the point at which the line $t = 60$ crosses the curve marked 18 lies on the $PE-t$ graph for the station. This curve may be drawn in and the unadjusted value of PE corresponding to each monthly temperature may be read off. The value of I to which this curve corresponds need not be known but it can obviously be estimated by comparison with the curves given, whose I values are marked along the line $PE = 1.6\text{ cm}$.

The final step of adjusting PE values for variations in the duration of daylight is then necessary.

Conclusions

The purpose of the present paper is not to attempt an assessment of the results which Thornthwaite's method yields but to simplify the procedure so that interested workers may more easily examine the results.

Inspection of the results in simple graphical form suggests that much of the elaboration of calculation results only in differences too small to have any significance. The use of a complicated formula for I gives results which, especially at higher temperatures, do not differ significantly from those which would be obtained by simpler formulae.

Some of the more obvious criticisms of the method have been anticipated by Thornthwaite who holds that, unexpected as are some of the conclusions implied by the formulae, the facts of observation compel him to accept them. See, for instance, 1944, p. 698; 1948, p. 90. We are, however, entitled to inquire whether the facts of observation cover a range of conditions sufficiently wide to establish the general truth of the conclusions which follow from them. The basic assertion of the whole

method is that water-need may be computed from temperature figures alone. This is not a claim that temperature is the only factor involved, but does imply that the other factors which control water-need correlate so highly with temperature that their net effect can be forecast by a consideration of temperature alone. Whether this is true for any part of Australia remains to be proved, but it is easy to find examples in which Thornthwaite's formulae give results which suggest that the effect of humidity on water-need is not satisfactorily dealt with by using temperatures alone. Thus Merbein and Sydney in March should have, according to Thornthwaite's method, *PE* values of 9.2 and 9.3 cm. The tank evaporimeter readings are 15.5 and 8.9 cm. Admitting all the doubts about the interpretation of evaporimeter readings it is difficult to accept the Thornthwaite figures as nearer the truth about water-need at the two places.

Next, it seems reasonable to question how far we should apply Thornthwaite's method of using *PE* to calculate run-off. The potential evapotranspiration is, by definition, the water-loss which occurs by evaporation and transpiration when water is constantly available in optimum quantity. In humid areas and irrigation areas these conditions are met and the surplus for storage and run-off will be simply (water available - *PE*). The value of such a calculation when water available exceeds *PE* only occasionally is doubtful. The occasional presence of water in free supply cannot immediately call into being the same machinery of transpiration as was present in the areas for which the formula was devised. It seems unlikely that the very great success of Thornthwaite's methods in computing run-off in the Tennessee Valley will be repeated when they are applied to catchments in open, dry sclerophyll forest.

Finally, the Australian geographer will be particularly interested in the proposal to make computed values of *PE* basic to a world classification of climate. It would appear that the method faces a logical difficulty which will be serious in much of Australia. The only temperatures available for the calculation of *PE* are those observed under existing conditions, viz. with water in free supply for a few months only and with sparse vegetative cover. The computed values of water-need will therefore not be what they would be if water-need were fully met and the temperatures were modified as a result.

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Appendix

Write mean monthly temp. $t = m + d$.

Where $m^{\circ}\text{C}$ is mean annual temperature

$d^{\circ}\text{C}$ (+ve or -ve) is departure from mean.

Monthly heat index $i = (t/5)^{1.514}$

$$= m/5^{1.514} (1 + d/m)^{1.514}$$

First factor = i_m , monthly heat index for a month with a mean temperature equal to annual mean.

Where $d < m$ (i.e. no month has a mean temperature below freezing point) second factor may be expanded as a convergent series

$$i = i_m (1 + 1.514 d/m + .39(d/m)^2 - .063 (d/m)^3 \dots)$$

Annual heat index $I = \Sigma i$

From definition of mean $\Sigma d/m = 0$

$$I = i_m (12 + .39 \Sigma (d/m)^2 \dots)$$

Σd^2 varies with the form of the temperature curve. The simplest analytical curve with the same general form as the usual annual temperature curve is the sine curve. Departures from this have been studied, for instance, by Köppen but not all differences affect Σd^2 .

For a temperature curve which is sinusoidal

$$\Sigma d^2 = 1.5 \Sigma r^2$$

where $r^\circ\text{C}$ is difference between temperatures of hottest and coldest months.

Suggested rule is thus:

$$I = i_m(12 + .6 r^2/m^2)$$

References

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